and outer radius, respectively, of a porous wall, m; m, flow rate of coolant, kg/(m·sec); j, flow rate of coolant, kg/(m²·sec); c_p, heat capacity of the coolant, J/(kg·deg); P, pressure, N/m²; P_0, P_1, pressure of coolant in front of the porous wall in the cold and hot state, respectively, N/m²; P_2, pressure of the coolant at the outlet from the porous wall, N/m²; λ_w, thermal conductivity of the material of the wall, W/(m·deg); γ, $(mc_p)/(2\pi\lambda_w)$; A (for $\xi_{T_0} = \xi_{\bar{T}}$) $- (P_1^2 - P_0^2)/(P_0^2 - P_2^2)$; α, coefficient of linear expansion, 1/deg; E, Young's modulus, kgf/mm²; μ, Poisson ratio; $\sigma_T(T)$, yield strength, kgf/mm²; σt_r, σt_θ, σt_z, radial, tangential, and axial thermal stresses, respectively, kgf/mm².

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STRUCTURAL AND MECHANICAL PROPERTIES AND EFFECTIVE PERMEABILITY OF FISSURED MATERIALS

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The article investigates the dependences of porosity and of the permeability tensor of an elastic fissured medium on the characteristics of its state of stress and the pressure of the permeating liquid.

Problems of permeation and convective transfer in deformed fissured and fractured-porous media are of interest in connection with the opening up of oil, gas condensate, and water deposits belonging to these types of collectors, and also for a number of problems of mining thermophysics. The structural method of describing motion in such media is based on the notions of continuity submitted in [1], and it has received fairly widespread application (see, e.g., [2-5]). Some inaccuracies characteristic of the continuous model in [2-5] were eliminated in [6]. In accordance with the model, the initial fractured-porous medium may be regarded as superposition of two coexisting porous continua modeling a system of interrelated cracks and a system of porous blocks.

The equations of motion in the mentioned continua contain as parameters fully characterizing the averaged properties of the medium: the effective porosity and permeability tensors referred to these continua, and also magnitudes describing the exchange of liquid between them. In regard to its meaning, the problem of determining the above parameters, which in the solution of various problems of permeation have to be regarded as known functions of the state of a medium filled with liquid, is completely analogous to the known rheological problem of the hydrodynamics of suspensions or the problem of determining the mechanical, thermophysical, and

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electrophysical properties of heterogeneous materials. However, an important feature of the structural and mechanical properties of fractured-porous media is that they very strongly depend on the pressure of the permeating liquid as well as on the state of stress of the medium, in particular, under certain conditions it is possible that the fissures close up and flow through them ceases practically completely. This circumstance was apparently taken into account for the first time by the condition of smooth closure of the walls of fissures suggested by Khristianovich in the theory of hydraulic rupture of beds (see [5]); the strong effect of the possible closure of fissures on the process of permeation was noted in [7]. A significant example of the quantitative description of this effect for steady-state permeation toward a borehole or gallery is contained in [8].

The effective characteristics of fissured or fractured-porous rocks can be determined experimentally with the aid of investigations of the corresponding core samples [9], from the curves of pressure recovery in the vicinity of boreholes [10], from the characteristics of the process of pumping finite amounts of liquid into boreholes [11], by observing the productivity of boreholes [12], with the aid of different complex investigations of permeation [13, 14], and also by seismic and geophysical methods [15]. However, regardless of some attempts in the literature at a theoretical generalization (see, e.g., [15-18]), an acceptable theory of the structural and mechanical properties of the media under investigation does not exist so far, and for the correlation of experimental data and the solution of actual problems of permeation empirical formulas are usually used in which the dependence of these magnitudes on the pressure in the fissues is taken into account but, as a rule, the effect of the state of stress of the medium on them is altogether ignored. Therefore, the suggested power formulas [3] and exponential formulas [2, 3, 9, 12] can apply, strictly speaking, only to media under conditions of hydrostatic pressure.

The following formulas are used particularly frequently:

$$k = k_0 \exp[a_k(p - p_0)], \quad m = m_0 \exp[a_m(p - p_0)]; \tag{1}$$

they correspond to the additional assumption that the rock pressure is constant; here, p_0 is the characteristic value of the formational pressure p, and the coefficients a_k and a_m are usually of the order of 10^{-8}-10^{-7} Pa^{-1}.

Below we investigate fissure porosity and permeability on the basis of the general model of [6] for media with plane fissures having smooth walls without protrusions and fractures preventing their closure, on the assumption that the medium is elastic but that the stresses in it and the pressure of the liquid in the fissures are arbitrary. If the permeability by blocks is much smaller than the permeability by fissures, then the obtained results have to approximately correct for fissured as well as for fractured-porous materials.

The opening (minor semiaxis) of a plane ellipsoidal fissure with radius c can be represented in the form [6, 19]

$$h = Ac(p - \mathbf{n}\sigma\mathbf{n})\,Y(p - \mathbf{n}\sigma\mathbf{n}), \tag{2}$$

where Y(x) is a Heaviside function, and the constant A depends in a known manner on the elastic properties of the monolithic or porous blocks separated by fissures. In accordance with the Boussinesq–Poiseuille law, the hydraulic conductivity of fissures is proportional to h^3, and their volume is proportional to h. If the gradient of the mean pressure in the fissures is $\nabla p = -\Sigma A_i e_i$, then the flow of liquid due to the i-th component ∇p is proportional to the vector $A_i h^3(\mathbf{n} \times e_i) \times \mathbf{n}$ [6]. The mean flow is obtained by averaging the fissures by their distribution function according to orientations and dimensions. According to the assumption the magnitude of c does not depend on p and σ, i.e., averaging with respect to c is trivial, and it suffices if we confine ourselves to averaging with respect to the vector $\mathbf{n}$, as if all the fissures had the same dimensions. If we determine in the ordinary manner the components of the permeability tensor through the proportionality factors between the corresponding components of the liquid flow and of the pressure gradient, we obtain, with a view to (2), that

$$k_{ij} = B \int [(\mathbf{n} \times \mathbf{e}_i) \times \mathbf{n}]_j (p - \mathbf{n}\sigma\mathbf{n})^3\, Y(p - \mathbf{n}\sigma\mathbf{n})\, f(\mathbf{n})\, d\mathbf{n}. \tag{3}$$

Analogously, for fissured porosity we have

$$m = C \int (p - \mathbf{n}\sigma\mathbf{n})\, Y(p - \mathbf{n}\sigma\mathbf{n})\, f(\mathbf{n})\, d\mathbf{n}. \tag{4}$$

The coefficients B and C contained in (3) and (4) do not depend on p, σ and they can be expressed through some reference values of permeability and porosity, and also of the pressure in the fissures and the characteristic compressive stress. In the past, in theoretical works, the authors used various types of averaging with respect to volume [15-18] instead of averaging with respect to distribution function $f(\mathbf{n})$ (which, for the sake of determinacy, we regard as normalized to unity).

A simpler situation arises in hydrostatic pressure on a macroscopically homogeneous and isotropic medium with stress σ or a medium in which all the stresses are oriented in one plane and which is subjected to arbitrary compression with $\sigma = \mathbf{n}\sigma\mathbf{n}$. In both cases [6]

$$k_{ij} = k\delta_{ij}, \quad k = k_0\left(\frac{p-\sigma}{p_0-\sigma}\right)^3, \quad m = m_0\frac{p-\sigma}{p_0-\sigma}, \quad p > \sigma. \tag{5}$$

In the general case of heteroaxial loading and arbitrary distribution of fissures (the medium is macroscopically homogeneous but not isotropic), it is natural to take as axes of coordinates the principal axes of the stress tensor. Then $\mathbf{n} = \{\cos\theta, \sin\theta\cos\varphi, \sin\theta\sin\varphi\}$, where θ and φ are the polar and azimuthal angle, respectively, of the spherical system of coordinates,

$$\begin{gathered}\alpha_{11} = \sin^2\theta, \quad \alpha_{12} = -\frac{1}{2}\sin 2\theta\cos\varphi, \quad \alpha_{13} = -\frac{1}{2}\sin 2\theta\sin\varphi, \\ \alpha_{22} = 1 - \sin^2\theta\cos^2\varphi, \quad \alpha_{33} = 1 - \sin^2\theta\sin^2\varphi, \\ \alpha_{23} = -\frac{1}{2}\sin^2\theta\sin 2\varphi \quad (\alpha_{ij} \equiv [(\mathbf{n}\times\mathbf{e}_i)\times\mathbf{n}]_j), \\ p - \mathbf{n}\sigma\mathbf{n} = p - \sigma_3 - (\sigma_1 - \sigma_3)\cos^2\theta - (\sigma_2 - \sigma_3)\sin^2\theta\cos^2\varphi\end{gathered} \tag{6}$$

and $d\mathbf{n} = \sin\theta d\theta d\varphi$ in (3) and (4).

Let us first examine the most important case as regards application, where $\sigma_1 = \sigma_2 + r > \sigma_2 = \sigma_3$ $(r = \sigma_1 - \sigma_2)$ and the last relation in (6) is written as follows:

$$p - \mathbf{n}\sigma\mathbf{n} = r(\psi - \cos^2\theta), \quad \psi = r^{-1}(p - \sigma_2). \tag{7}$$

Since this magnitude is the argument of the Heaviside function in (3) and (4), its positive value determines the region of integration in these relations. If $\psi > 1$ $(p > \sigma_1)$ or $\psi < 0$ $(p < \sigma_2)$, then all the fissures are open or closed, respectively. When $0 < \psi < 1$, only the fissures are open for which $\theta^* < \theta < \pi - \theta^*$, where the critical value of the angle is $\theta^* = \arccos\sqrt{\psi}$.

When $\sigma_2 - r < \sigma_2 = \sigma_3$ $(r = \sigma_2 - \sigma_1)$, we have instead of (7)

$$p - \mathbf{n}\sigma\mathbf{n} = r(\cos^2\theta - \psi), \quad \psi = r^{-1}(\sigma_2 - p). \tag{8}$$

In this case all the fissures are open or closed when $\psi < 0$ or $\psi > 1$, respectively, and when $0 < \psi < 1$, only those fissures are open for which $0 < \theta < \theta^*$ or $\pi - \theta^* < \theta < \pi$, where $\theta^* = \arccos\sqrt{\psi}$.

In the general case, when the state of stress of the medium is not axisymmetric, the shape of the regions of integration with respect to the variables θ and φ in (3) and (4) is more complex and can be determined from (6).

Thus the determination of fissure permeability and fissure porosity within the framework of the model under examination reduces in essence to the calculation of the integrals in (3) and (4) by using relations (6); this can be effected numerically for material with arbitrary distribution of fissures in regard to orientation. For the sake of simplicity we confine ourselves to the analysis of macroscopically axisymmetric materials; their anisotropy is described on the whole by specifying the unit vector $\lambda = \{\cos\theta^\circ, \sin\theta^\circ\cos\varphi^\circ, \sin\theta^\circ\cdot\sin\varphi^\circ\}$, which determines the direction of the axis of symmetry. Since the directions n and −n are equivalent, f(n) may be regarded as an even function of the unit scalar argument λn. If we expand this function into a series of powers of λn and confine ourselves to the first two terms of the expansion, we have, with a view to the conditions of normalization,

$$f(\mathbf{n}) = \frac{1}{4\pi(1+\eta)}[1 + 3\eta(\mathbf{n}\lambda)^2]. \tag{9}$$

Here the coefficient η acts as "parameter of order" characterizing the degree of anisotropy. The distribution function (9) may be regarded as a fairly good approximation for describing any axisymmetric material.

In the case $\sigma_1 > \sigma_2 = \sigma_3$ calculations using (6) and (7) lead to the following analytical representations for k_{ij} and m:

$$k'_{11} = a\{J_2 + (3\eta/2)[J_3 + (2J_2 - 3J_3)\cos^2\theta^\circ]\},$$

$$k'_{22} = a\{J_1 - (1/2)J_2 + (3\eta/2)[J_2\cos^2\theta^\circ + (1/4)(3 - 7\cos^2\theta^\circ -$$

$$- 2\sin^2\theta^\circ\cos^2\varphi^\circ)J_3]\}, \quad k'_{33} = a\{J_1 - (1/2)J_2 + (3\eta/2)[J_2\cos^2\theta^\circ + (1/4)(1 - 5\cos^2\theta^\circ + 2\sin^2\theta^\circ\cos^2\varphi^\circ)J_3]\}, \tag{10}$$

$$k'_{12} = -a(3\eta/2)(J_2 - J_3)\sin 2\theta^\circ\cos\varphi^\circ,$$

$$k'_{13} = -a(3\eta/2)(J_2 - J_3)\sin 2\theta^\circ\sin\varphi^\circ,$$

$$k'_{23} = -a(3\eta/8)J_3\sin^2\theta^\circ\sin 2\varphi^\circ, \quad a = \frac{r^3}{2(1+\eta)}, \quad \mathbf{k}' = \frac{\mathbf{k}}{B},$$

$$m = \frac{r}{2(1+\eta)}\left\{J_4 + \frac{3}{2}\eta[J_5 + (2J_4 - 3J_5)\cos^2\theta^\circ]\right\}, \quad m' = \frac{m}{C},$$

where J_i depends on ψ. When $0 < \psi < 1$, we have

$$\begin{gathered} J_1 = 0{,}92\,\psi^{7/2}, \quad J_2 = 0{,}92\,\psi^{7/2} - 0{,}10\,\psi^{9/2}, \\ J_3 = 0{,}92\,\psi^{7/2} - 0{,}20\,\psi^{9/2} + 0.02\,\psi^{11/2}, \\ J_4 = 1.34\,\psi^{3/2}, \quad J_5 = 1.34\,\psi^{3/2} - 0.22\,\psi^{5/2} \end{gathered} \tag{11}$$

and when $\psi > 1$,

$$\begin{gathered} J_1 = 2\psi^3 - 2\psi^2 + 1.20\,\psi - 0.28, \quad J_2 = 1.34\,\psi^3 - 0.80\,\psi^2 + \\ + 0{,}34\,\psi - 0.06, \quad J_3 = 1.06\,\psi^3 - 0.46\,\psi^2 + 0{,}16\,\psi - 0{,}02, \\ J_4 = 2\psi - 0.66, \quad J_5 = 1.34\,\psi - 0.22. \end{gathered} \tag{12}$$

Thus, in the case under consideration, fissure permeability and fissure porosity depend, first, on the magnitude of θ°, φ°, and η, characterizing the properties of the medium itself in the unloaded state and, second, on the parameters $r = \sigma_1 - \sigma_2$ and $\psi = r^{-1}(p - \sigma_2)$ characterizing the state of stress and the pressure in the fissures. Formulas (10) become greatly simplified for macroscopically isotopic media ($\eta = 0$) when the crossed components of the permeability vector vanish. These formulas are simplified, but the permeability tensor has diagonal form even when the axes of symmetry of the anisotropic medium and its state of stress coincide ($\theta^\circ = 0$).

In the case $\sigma_1 < \sigma_2 = \sigma_3$, formulas (10) are, as before, correct for m and the component of the tensor **k**, but the functions J_i they contain have to be replaced by J'_i, where in the interval $0 < \psi < 1$

$$J'_i = J_i - \Omega_i, \tag{13}$$

where J_i are determined in (11), and when $\psi < 0$,

$$\begin{gathered} J'_i = -\Omega_i, \quad \Omega_1 = 2(\psi^3 - \psi^2 + 0.60\,\psi - 0.14), \\ \Omega_2 = 2(0.67\,\psi^3 - 0.40\,\psi^2 + 0.17\,\psi - 0.03), \\ \Omega_3 = 2(0.53\,\psi^3 - 0.23\,\psi^2 + 0.08\,\psi - 0.01), \\ \Omega_4 = 2(\psi - 0.33), \quad \Omega_5 = 2(0.67\,\psi - 0.13). \end{gathered} \tag{14}$$

The nature of the dependence of the magnitudes $\ell_{ij} = 2k_{ij}r^{-3}$ and $n = 2mr^{-1}$ on various parameters for states of axisymmetric stress of macroscopically axisymmetric material, de-

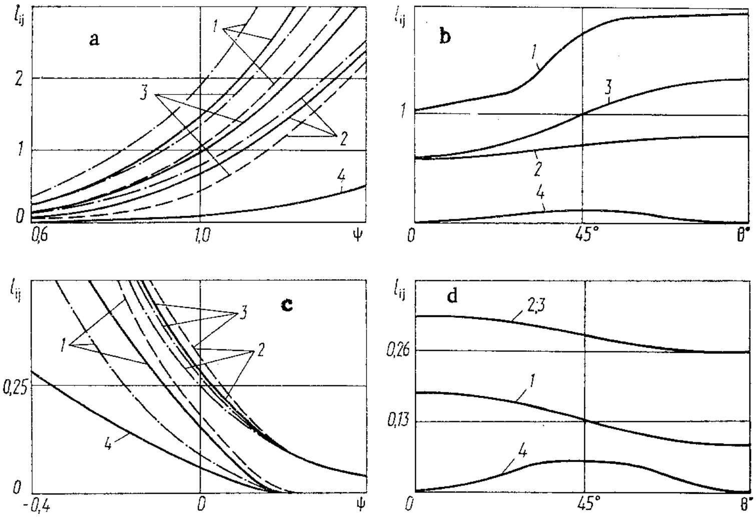

Fig. 1. Reduced coefficients of permeability with state of axisymmetric stress of axisymmetric material in the cases $\sigma_1 > \sigma_2 = \sigma_3$ (a, b) and $\sigma_1 < \sigma_2 = \sigma_3$ (c, d) for $\varphi° = 0$, $\eta = 1$: 1-4) ℓ_{11}, ℓ_{22}, ℓ_{33}, $-\ell_{12}$; a, c) $\theta° = 0$, 45, 90° (dashed, solid, and dot-dash curves, respectively); b, d) $\psi = 1$.

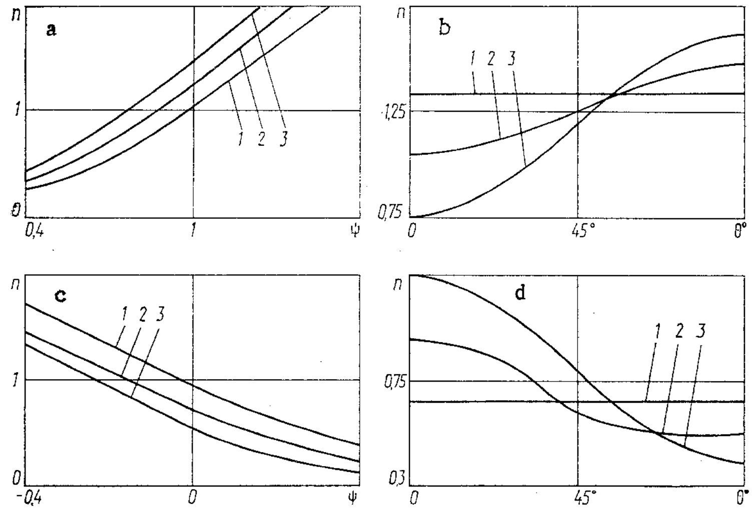

Fig. 2. Reduced porosity of axisymmetric material with state of axisymmetric stress in the cases $\sigma_1 > \sigma_2 = \sigma_3$ (a, b) and $\sigma_1 < \sigma_2 = \sigma_3$ (c, d) for $\varphi° = 0$; a, c) $\eta = 1$, $\theta° = 0$ (1), 45° (2), 90° (3); b, d) $\psi = 1$, $\eta = 0$ (1), 1 (2), ∞ (3).

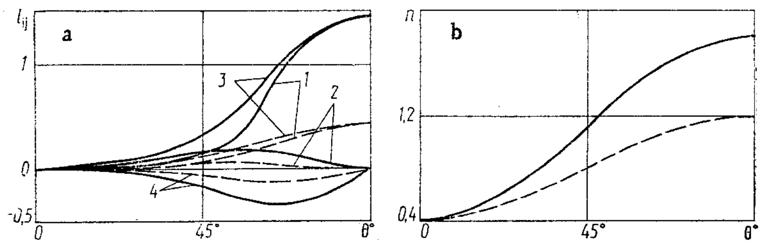

Fig. 3. Reduced coefficients of permeability (a) and porosity (b) for materials with uniformly oriented fissures with state of nonaxisymmetric stress; 1-4) ℓ_{11}, ℓ_{22}, ℓ_{33}, ℓ_{12}; $(\sigma_2 - \sigma_3)(\sigma_1 - \sigma_3)^{-1} = 0.3$ (solid) and 0.6 (dashed curves); $\varphi_0 = 0$, $\psi = 1.2$.

scribed by the distribution (9), is shown in Figs. 1 and 2 (all the magnitudes in the figures are dimensionless). Particularly important is the very strong dependence of these magnitudes on the parameter ψ characterizing (in relative units) the pressure of the liquid in the fissures. It is not difficult to obtain asymptotically (for large p) from (10) the cubic dependences for ℓ_{ii} and the linear dependence for n, all of them corresponding to (5). The crossed components ℓ_{ij} $(i \neq j)$ vanish when $\eta = 0$ or $\theta° = 0, \pi/2$.

If the state of stress is not axisymmetric, then numerical methods are needed for integration in (3) and (4), even in the analysis of isotropic and simple axisymmetric materials. An exception are materials in which all cracks are oriented alike (i.e., lie in one plane); this is quite characteristic of sedimentary rocks, where instead of (9) we have

$$f(\mathbf{n}) = \delta(\mathbf{n} - \mathbf{n}_0), \quad \mathbf{n}_0 = \{\cos\theta_0,\ \sin\theta_0\cos\varphi_0,\ \sin\theta_0\sin\varphi_0\}. \tag{15}$$

In this case, putting $\sigma_1 > \sigma_2 > \sigma_3$ for the sake of determinacy, we have

$$k_{ij} = \alpha_{ij}(\theta_0,\ \varphi_0)\,\omega^3(\theta_0,\ \varphi_0), \quad m = \omega(\theta_0,\ \varphi_0),$$
$$\omega = p - \sigma_3 - (\sigma_1 - \sigma_3)\cos^2\theta_0 - (\sigma_2 - \sigma_3)\sin^2\theta_0\cos^2\varphi_0, \tag{16}$$

where α_{ij} are determined in (6) in the region of positiveness of the function ω; when $\omega < 0$, all the fissures are closed, $k_{ij} = 0$, $m = 0$. The dependences (16) are presented in Fig. 3.

For $\sigma_1 > \sigma_2 > \sigma_3$ and f(n) from (9), a cycle of numerical calculations of the porosity and permeability coefficients of fissured materials was carried out. These data are too cumbersome to present here but we will show that the nature of their dependence on the examined parameters is the same as in Figs. 1-3. Investigation of the properties of anisotropic nonaxisymmetric materials on the basis of the suggested theory also requires in most situations numerical integration in (3), (4).

A comparison of the obtained results with experimental data is made difficult by the fact that, as a rule, experimental works do not contain information on the distribution of fissures in respect of orientation in the investigated samples, and the data obtained in laboratory experiments are very often of a qualitative nature only; this was admitted, e.g., in [13]. The above comparison is objectively somewhat superficial, and at present it makes sense to compare the theoretical formulas with the empirical ones, regarding the latter as some generalization of variegated experiments, but not directly with the data of original experiments. Thus, confining ourselves to the analysis of macroscopically isotropic materials in the state of hydrostatic compression, we compare the theoretical formula for permeability from (5) with the empirical formula in (1), representing them in dimensionless form:

$$\frac{k}{k_0} = \exp(-\alpha x), \quad \alpha = a_k p_0, \quad x = \frac{p_0 - p}{p_0}, \tag{17}$$

$$\frac{k}{k_0} = \left(1 - \frac{x}{\beta}\right)^3, \quad 0 \leqslant x \leqslant \beta = 1 - \frac{\sigma}{p_0} < 1. \tag{18}$$

In the literature it is admitted that the coefficient a_k (and consequently also α) is bound to depend on the compressive stress σ, but the form of this dependence is not being determined. It is therefore expedient to compare the families of curves (17) and (18), regarding α and β as variable parameters (see Fig. 4). It can be seen from the figure that with sufficiently large α (in particular those corresponding to anomalously high formational pressures [3], when fissure permeability is in fact large) it becomes possible that there is fairly good agreement between the curve (17) and any of the curves (18) corresponding to a certain value of β depending on α. To a certain extent this circumstance explains theoretically the efficiency of the empirical exponential formula in (1). Comparison of the curves (17) and (18) also makes it possible to determine the approximate dependence of the empirical coefficient α on the stress σ.

With random isotropic distribution of fissures, fissued material is equivalent to the radially homogeneous sandstone dealt with in [20]. In this case, in accordance with (5), the theoretical coefficient of compressibility of the pores

$$a_m = (p - \sigma)^{-1} \tag{19}$$

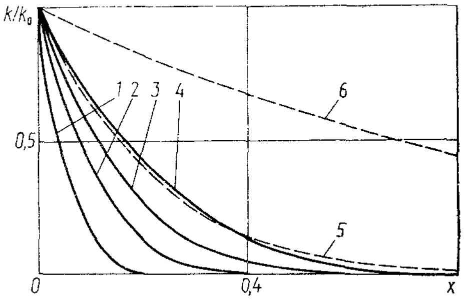

Fig. 4. Comparison of the theoretical formula (17) (dashed) and the empirical formula (18) (solid curves) for scalar permeability: β = 0.2 (1), 0.4 (2), 0.6 (3), 0.8 (4); α = 5 (5), 1 (6).

differs considerably from $a_m \sim p^{-1/3}$ for the models of elastic porous and fissured rocks discussed in [12]. However, a number of experiments lead directly in particular to the form (19) of the coefficient of compressibility [12]; this is in contradiction to the presented models, but it confirms the theory of [12]. We note that the magnitude of the coefficient of compressibility is very important in approximate evaluations of oil reserves by the method of the elastic material balance on the basis of data of exploratory tests. Without going into details, we will show that when a_m from (19) is used in the corresponding calculations, it leads to results which differ by a multiple from the results obtained on the basis of other models of fissured reservoirs in [12].

Some indirect testimony to the correctness of the developed theory could be quoted. It is known, e.g., that the axial permeability of cylindrical samples subjected to axial pressure σ_1 decreases with increasing lateral pressure σ_2 [12, 16]. For the sake of simplicity we take isotropic materials (η = 0), for the coefficient of axial permeability we obtain from (10) that $k_{11} = 0.5J_2r^3$, and then for $\sigma_1 \geqq p > \sigma_2 = \sigma_3$ we have

$$\frac{dk_{11}}{d\sigma_2} = \frac{r^2}{2}\left(-3J_2 + \frac{p-\sigma_1}{r}\,\frac{dJ_2}{d\psi}\right) < 0.$$

It is not difficult to show that the sign of this derivative is retained also for $p > \sigma_1$, as well as for $\eta \neq 0$ in (9), which corresponds to anisotropic axisymmetric materials.

In conclusion, we emphasize that the results obtained above make it possible to state and solve various problems of permeation in fissured reservoirs. In fractured-porous media there may also be considerable permeability through porous blocks which is also bound to depend on the degree of opening of the fissures, i.e., on the pressure in the fissures and the state of stress of the medium. Determination of block permeability is an independent problem that has not yet been solved.

NOTATION

A, B, C, constant coefficients; a_k, a_m, coefficients in (1); c, radius of fissures; f(n), distribution function of cracks in respect to orientation; e_i, unit vector; h, fissure opening; J_i, J'_i, functions determined in (11)-(14); k_{ij}, k'_{ij}, ℓ_{ij}, true and reduced permeability tensors, respectively; m, m', n, porosity and its reduced values; $\mathbf{n}$, unit vector of the normal to the plane of the fissure; p, pressure in the fissures; $r = |\sigma_1 - \sigma_2|$; x, dimensionless pressure introduced into (17); α, β, coefficients in (17), (18); $\{\alpha_{ij}\}$, tensor introduced in (6); η, parameter of order; θ, φ, angular coordinates; ψ, variable determined in (7) or (8); Ω_i, functions determined in (14); ω, function from (16).

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ISOTHERMAL FLUID FLOW IN A PACKING OF SPHERES

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Simultaneous measurements of the velocity profiles inside and behind a packing made of spheres are used to establish the pattern of isothermal flow of a fluid inside the packing.

Several investigations have found that the velocity field after a granular bed may be quite different from the velocity field inside the bed [1, 2]. We therefore made use of studies which determined the fluid velocity inside the bed. Analysis of these works showed that all of the contact methods of measurement give a planar or nearly planar velocity profile if the size of the transducer is comparable to or greater than the size of a granule of the porous medium [2-4].

It has been established by all of the noncontact and diffusive methods of determining velocity that the fluid velocity is significantly higher near the wall than in the center of the packing [5-7]. This discrepancy in the findings is evidently due to the fact that in the measurement of velocity from the heat and mass transfer from a transducer comparable in size to the granule size in the bed, one is actually determining the hydrodynamic situation

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